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EQUILIBRIUM HYDRODYNAMIC VARIABLES BEHIND A REFLECTED SHOCK WAVE IN HYDROGEN

EUGENE B. TURNER

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⑩ by

Eugene B. Turner

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PHYSICAL RESEARCH LABORATORY
Space Technology Laboratories, Inc.
P. O. Box 95002
Los Angeles 45, California

ABSTRACT

Hydrodynamic variables such as density ratio, pressure ratio, temperature, fraction of dissociation and ionization and the reflected shock velocity have been calculated for a reflected shock wave in hydrogen for one dimension. The data are presented as families of curves of these variables plotted against the initial shock Mach number for a range of unity to 200 with the initial pressure as a parameter.

Hydrodynamic equations are derived which include the effects of dissociation, ionization and atomic excitation. Reference is made to a previous report for the hydrodynamic variables behind the initial shock wave in hydrogen which were used in these calculations.

Deviations from the so called "strong shock approximation" are quite large, especially in the regions where the gas is being dissociated or ionized. The density and pressure ratios across the reflected shock wave may be two to three times greater with a corresponding decrease in the reflected shock velocity and the temperature.

1. INTRODUCTION

The hydrodynamic variables for a primary shock wave moving into hydrogen gas at room temperature (300° K) have been calculated previously and the results were published in a Space Technology Laboratories report¹. These data allow one to determine quickly such quantities as the degree of ionization, the density ratio, the temperature, etc., from a measurement of the shock velocity².

Very often one wishes also to know the variables behind the reflected shock at the end of a shock tube. Therefore, using the results of the above-mentioned report, the calculations have been extended to include the hydrodynamic variables behind the reflected shock. The variables calculated were the pressure ratio across the reflected shock, the density ratio, the temperature, the fraction dissociated or ionized and the reflected shock velocity. The latter is very useful because the theoretical reflected shock velocity can be compared with that measured experimentally to determine whether the gas is behaving according to theory -- in particular, whether equilibrium has been established.

These calculations are for one-dimensional flow only, and, as implied in the title, equilibrium has been assumed throughout. The range covered in this report is from a shock Mach number, M_s , of unity up to 200. Above $M_s = 200$ the variables are given with sufficient accuracy by asymptotic values.

¹ Turner, E. B., "Equilibrium Hydrodynamic Variables Behind a Normal Shock Wave in Hydrogen," STL report GM-TR-0165-00460, dated 26 August 1958.

² The hydrodynamic curves given in GM-TR-0165-00460 and in the present report apply also to the isotopes of hydrogen, namely, deuterium and tritium. The shock velocity must therefore be measured by a dimensionless variable, M_s , which is the shock velocity divided by the sound speed of the undisturbed gas.

II. THEORY

The calculations naturally divide into several regions depending upon the conditions of the gases behind the primary and reflected shock waves. The formulae and methods of calculation are different for the various regions. The regions are the following:

- (a) The gas behind the reflected shock can be considered ideal, $M_s = 1 - 5$.
- (b) The gas behind the reflected shock is partially dissociated, $M_s = 4 - 12$.
- (c) The gas behind the reflected shock is completely dissociated and the gas behind the primary shock is partially dissociated, $M_s = 12 - 18$.
- (d) The gas behind the reflected shock is partially ionized and the gas behind the primary shock is not completely dissociated, $M_s = 16 - 18$.
- (e) The gas behind the reflected shock is partially ionized, $M_s = 20 - 44$.
- (f) The gas behind the reflected shock is fully ionized and the gas behind the primary shock is still partially ionized, $M_s = 40 - 64$.
- (g) The gases behind both the reflected and primary shock waves are completely ionized, $M_s = 52 - 200$.
- (h) Above $M_s = 200$ asymptotic values can be used with sufficient accuracy.

There will, of course, be some overlapping of these regions. The notation used is as follows:

- p = pressure
- ρ = density
- T = temperature
- α = fraction dissociated or ionized, as the case may be
- u = flow velocity
- a = sound speed
- U = primary shock velocity
- U_R = reflected shock velocity
- h = specific enthalpy

Subscripts are used to designate a particular region of the flow:

- (1) - the undisturbed gas at room temperature
- (2) - the gas behind the primary shock wave
- (5) - the gas behind the reflected shock wave

Consider now a primary shock wave with velocity U hitting a solid wall and reflecting with a velocity U_R .

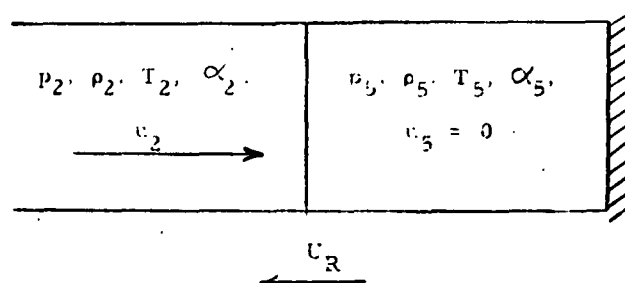


Fig. 1

The condition for reflection is that the gas behind the primary shock wave, moving with a velocity u_2 , be brought to rest, i.e., $u_5 = 0$.

One method of approach is to consider the more general problem of a shock wave moving in a tube with flow velocities on either side of the shock wave. Let the region ahead of the shock be denoted by the subscript (a) and the region behind the shock by (b).

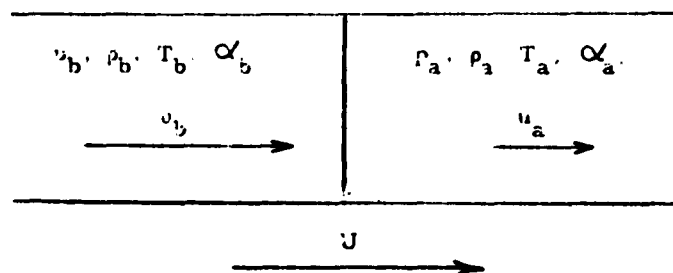


Fig. 2

Then the three conservation equations across the shock wave can be written:

$$\rho_b (U - u_b) = \rho_a (U - u_a) \quad \text{-- conservation of mass,} \quad (1)$$

$$\rho_b (U - u_b)^2 - \rho_a (U - u_a)^2 = p_a - p_b \quad \text{-- conservation of momentum,} \quad (2)$$

$$\frac{1}{2} (U - u_a)^2 + h_a = \frac{1}{2} (U - u_b)^2 + h_b \quad \text{-- conservation of energy.} \quad (3)$$

From these three equations can be derived the following relation for the flow velocities,

$$(u_b - u_a)^2 = \frac{p_a}{\rho_a} \frac{\left(\frac{\rho_b}{\rho_a} - 1\right)}{\frac{\rho_b}{\rho_a}} \left(\frac{p_b}{p_a} - 1\right). \quad (4)$$

Now let region (b) be region (5) and (a) be (2). Then

$$u_2^2 = \frac{p_2}{\rho_2} \frac{\left(\frac{\rho_5}{\rho_2} - 1\right)}{\frac{\rho_5}{\rho_2}} \left(\frac{p_5}{p_2} - 1\right). \quad (5)$$

To proceed further it is necessary to specify the condition of the gases on either side of the reflected shock wave. Consider first an ideal gas. Then the ratio ρ_5/ρ_2 is given by the Rankine-Hugoniot equation*.

$$\frac{\rho_5}{\rho_2} = \frac{\mu \frac{p_5}{p_2} + 1}{\frac{p_5}{p_2} + \mu}, \quad (6)$$

* See Reference 1.

where μ is given by $\mu = \gamma + 1/\gamma - 1$. The ratio p_2/ρ_2 can be written as

$$\frac{p_2}{\rho_2} = \frac{p_2}{p_1} \frac{p_1}{\rho_2} \frac{p_1}{p_1} = \frac{p_2}{p_1} \frac{\frac{p_2}{p_1} + \mu}{\mu \frac{p_2}{p_1} + 1} \frac{1}{\gamma} a_1^2,$$

using the Rankine-Hugoniot equation for p_2/ρ_1 and the expression for the sound speed, $a^2 = \gamma (p/\rho)$. The flow velocity behind the primary shock wave in an ideal gas is

$$\left(\frac{u_2}{a_1}\right)^2 = \frac{(\mu - 1)^2 \left(\frac{p_2}{p_1} - 1\right)^2}{(\mu + 1) \left(1 + \mu \frac{p_2}{p_1}\right)}. \quad (7)$$

This equation follows from equation (4) and equation (6) applied to the primary shock wave.

Putting these into equation (5) gives the quadratic equation,

$$\frac{\left(\frac{p_2}{p_1} - 1\right)^2}{\frac{p_2}{p_1} \left(\frac{p_2}{p_1} + \mu\right)} = \frac{\left(\frac{p_5}{p_2} - 1\right)^2}{\left(\mu \frac{p_5}{p_2} + 1\right)} \quad (8)$$

which has only one physically possible root,

$$\frac{p_5}{p_2} = \frac{(\mu + 2) \frac{p_2}{p_1} - 1}{\mu + \frac{p_2}{p_1}}. \quad (9)$$

The density ratio across the reflected shock is given by equation (6), and the temperature ratio is merely the pressure ratio, p_5/p_2 , divided by the density ratio, ρ_5/ρ_2 . From the conservation of mass across the reflected shock it follows that

$$U_R = \frac{u_2}{\frac{p_5}{p_2} - 1} \quad (10)$$

There are two ranges of shock velocity where the gas may be considered ideal. The first is the range of relatively weak shocks before the hydrogen starts dissociating. Then

$$\gamma = \frac{C_p}{C_v} = \frac{7}{5} \text{ so } \mu = 6.$$

The second is the range where the gases behind both the reflected shock wave and the primary shock wave are completely ionized and the enthalpy of the gas behind the primary shock is much larger than the energy of ionization. This range of shock velocities is from $M_s = 200$ to ∞ . For this case $\gamma = 5/3$ so $\mu = 4$. Then

$$\frac{p_5}{p_2} = 6.0$$

and

$$\frac{\rho_5}{\rho_2} = 2.5.$$

From equation (10)

$$\frac{U_R}{u_2} = \frac{2}{3},$$

but since

$$\frac{u_2}{U} = \frac{3}{4},$$

$$\frac{U_R}{U} = \frac{1}{2}.$$

The temperature

$$\frac{T_5}{T_2} = \frac{p_5}{p_2} \cdot \frac{\rho_2}{\rho_5} = 2.4.$$

These are the asymptotic values which are approached for very high shock velocities, and, as will be seen from the curves, they are, in most cases, sufficiently accurate above $M_s = 200$.

Next the calculations will be discussed for the regions where the gas behind the reflected shock wave is fully ionized or fully dissociated. The gas behind the primary shock can be partially or fully ionized or dissociated, respectively. This includes regions (c), (f) and (g) as outlined previously.

At this point it is necessary to consider the equation of state. It can be written in general as

$$p = \lambda \rho RT \quad (11)$$

For an ideal gas that is not yet dissociated $\lambda = 1$. For a partially dissociated gas $\lambda = (1 + \alpha)$ where α is the fraction dissociated, and for a partially ionized gas $\lambda = 2(1 + \alpha)$ where α is now the fraction ionized.

We will again make use of equation (5). The density ratio across the reflected shock is now given by a revised Rankine-Hugoniot equation,

$$\frac{\rho_5}{\rho_2} = \frac{4 \frac{p_5}{p_2} + 1}{\frac{p_5}{p_2} + 4 - 2 \frac{1 - \alpha_2}{1 + \alpha_2} \frac{\theta}{T_2}} \quad (12)$$

α_2 is the fraction ionized or dissociated and θ is the energy per atom of ionization* or the energy per molecule of dissociation respectively divided by the Boltzmann constant.

* Strictly one should subtract from $k\theta$ the average excitation energy per atom or per molecule in state (2). This was done for the case of partial ionization, but not for the case of partial dissociation where molecular excitation data was not available.

In order to find p_2/ρ_2 we note that

$$\frac{T_2}{T_1} = \frac{1}{\lambda} \frac{p_2}{\rho_2} \frac{\rho_1}{p_1} = \frac{1}{\lambda} \frac{p_2}{\rho_2} \frac{7}{5a_1^2}$$

so

$$\frac{p_2}{\rho_2} = \frac{5\lambda}{7} \frac{T_2}{T_1} a_1^2$$

Putting these into equation (5) results in the quadratic equation,

$$\frac{p_5^2}{p_2^2} - 2(2A + 1 - B) \frac{p_5}{p_2} - (A + 2B - 1) = 0, \quad (13)$$

where

$$A = \frac{7}{15\lambda} \left(\frac{u_2}{a_1} \right)^2 \frac{T_1}{T_2}$$

and

$$B = \frac{1}{3} \frac{1 - a_2^2}{1 + a_2^2} \frac{\theta}{T_2}$$

The values of variables such as T_2/T_1 , u_2/a_1 , and a_2 were taken from Reference 1. After solving this quadratic equation for p_5/p_2 , ρ_5/ρ_2 can be found from equation (12). The reflected shock velocity is given by equation (10).

The calculation of the variables behind the reflected shock for shock velocity ranges (b), (d) and (e) was somewhat more difficult as it involved an interpolation procedure. These are the ranges for which the gas behind the reflected shock is either partially ionized or partially dissociated. It is necessary to assume the state of the gas behind the reflected shock and then calculate the residual velocity. Two or three calculations are usually sufficient to give accurate interpolated values for the variables for zero velocity.

The equation for the velocity for the case of partial ionization or dissociation was derived in Reference 1 for the primary shock wave*, but with a slight generalization it can also be used for the reflected shock wave. For the reflected shock wave the equation can be written

$$\left(\frac{u_5 - u_2}{a_1}\right)^2 = \frac{5}{7} \lambda_2 \frac{T_2}{T_1} - 1 + \frac{\lambda_5}{\lambda_2} \frac{T_5}{T_2} + \sqrt{1 + \frac{\lambda_5}{\lambda_2} \frac{T_5}{T_2} + \frac{4(h_5 - h_2)^2}{\lambda_2 R T_2^2} - 4 \frac{(h_5 - h_2)}{\lambda_2 R T_2} \frac{\lambda_5 T_5}{\lambda_2 T_2} - 1} \quad (14)$$

The procedure is to first choose the enthalpy, h_5 . Then the pressure ratio across the reflected shock wave is determined by the relation,

$$\frac{p_5}{p_2} = \frac{h_5 - h_2 + \frac{1}{2} u_2^2}{\lambda_2 R T_2} \quad (15)$$

which is derived from the conservation conditions across a shock wave. From h_5 and p_5 one can now determine T_5 and λ_5 . This can be facilitated by using a family of curves of enthalpy versus temperature for several pressures and a family of curves of fraction ionized or dissociated versus temperature for the same pressures. The calculation of the enthalpy and fraction of ionization is explained in Reference 1, and the curves are shown in Figures 3 and 4 of this report.

Similar curves for the enthalpy, together with curves for the fraction of dissociation, are given in the Bureau of Standards Circular 564, 1 November 1955, entitled "Tables of Thermal Properties of Gases." These curves cover the region of dissociation while the curves in Figures 3 and 4 cover the region of

* This equation was originally derived by E. L. Resler in his Ph. D. thesis entitled "High Temperature Gases Produced by Strong Shock Waves." Cornell University, 1951.

temperatures where the hydrogen is partially ionized. The enthalpy for partially ionized hydrogen given in Figure 3 includes the energy of excitation of the neutral atoms. The Bureau of Standards' curves for the enthalpy of partially dissociated hydrogen, however, do not include the effect of molecular excitation of the undissociated molecules.

The values of the variables in region (2) such as u_2/a_1 , λ_2 , T_2 , h_2 and p_2 were taken from the first hydrodynamic report¹. Two or three estimates of h_5 must be made to give values of u_5 on either side of zero. Then the hydrodynamic variables h_5 , p_5/p_2 , T_5 and λ_5 can be interpolated for $u_5 = 0$. The density ratio is given by

$$\frac{\rho_5}{\rho_2} = \frac{p_5 \lambda_2 T_2}{p_2 \lambda_5 T_5},$$

and the reflected shock velocity is

$$U_R = \frac{h_5 - h_2}{u_2} - \frac{1}{2} u_2.$$

The hydrodynamic variables behind the reflected shock wave, calculated according to the methods given above, have been plotted on graphs shown in Figures 5 - 9. In Figure 5 the density ratio behind the primary shock wave has been plotted along with the density ratio behind the reflected shock wave so that the total density ratio can be quickly determined without reference to the first hydrodynamic report¹. In Figure 8 the temperature behind both the reflected and primary shock waves are shown.

These hydrodynamic curves show the large effect that dissociation and ionization have on the variables behind the reflected shock wave. An assumption of the asymptotic value of 2.5 for the density ratio, for example, would be too low by a factor of more than 2 over the range of $M_s = 6$ to 40. Similar large deviations occur in the pressure ratio across the reflected shock wave. The asymptotic value is 6, but the pressure ratio has maxima of more than 20. As one should expect, the reflected shock velocity becomes much less in the range where the pressure and density ratios are large. The minimum reflected shock velocity is less than one-third of the asymptotic value of $1/2 U$.

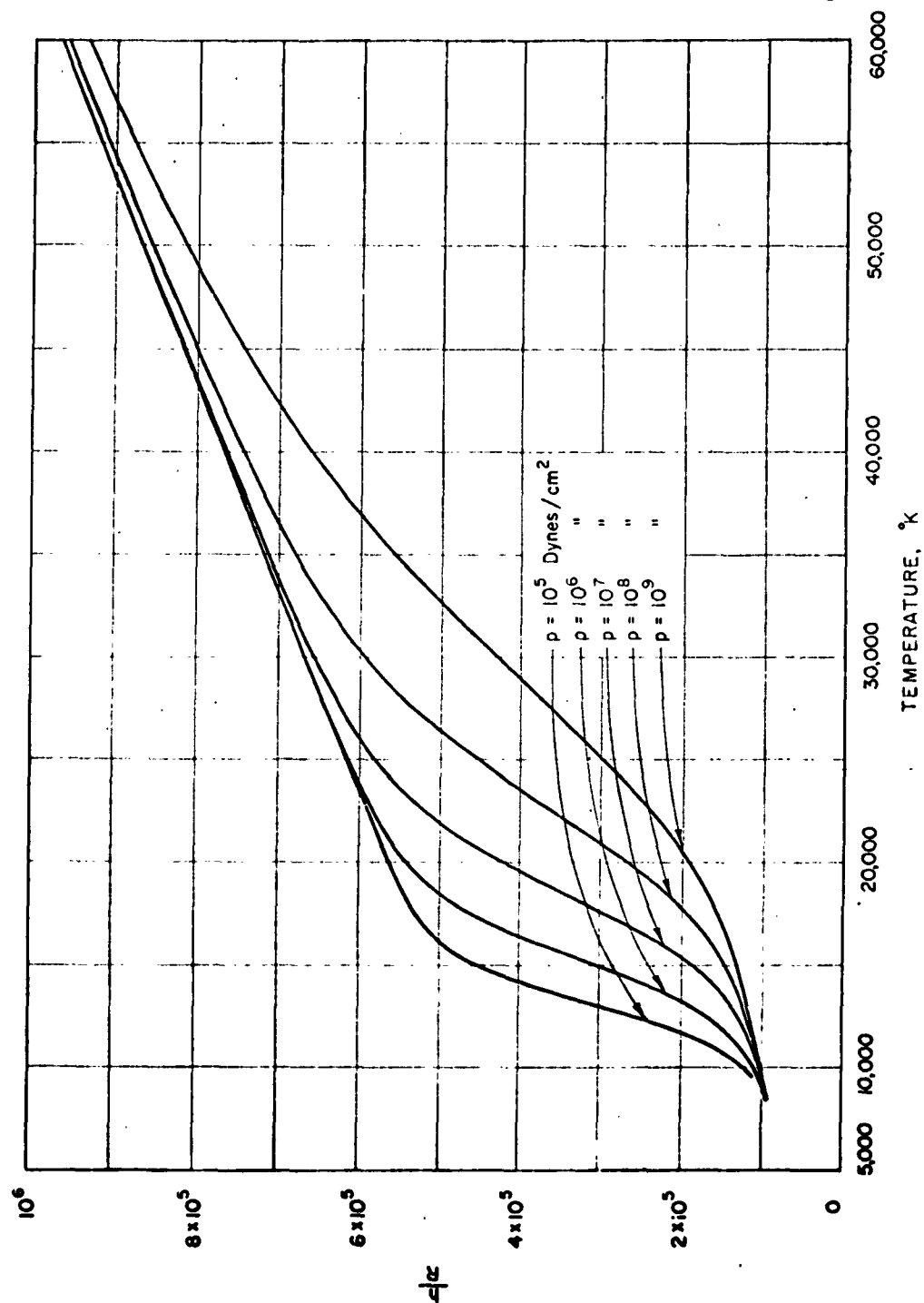


Fig. 3 Specific enthalpy of hydrogen vs temperature for several pressures

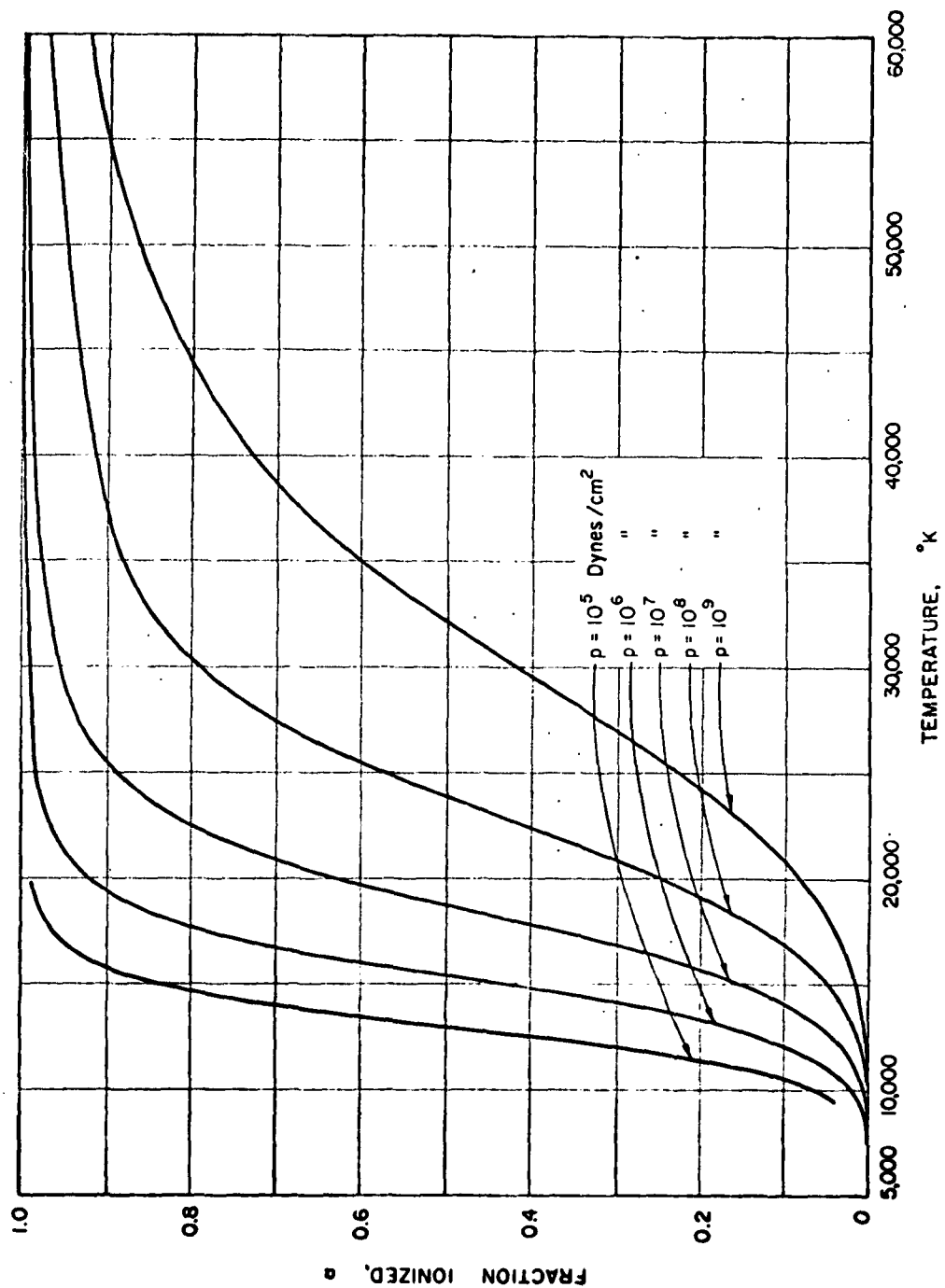


Fig. 4 Fraction of hydrogen ionized vs temperature for several pressures (calculated from the Saha equation).

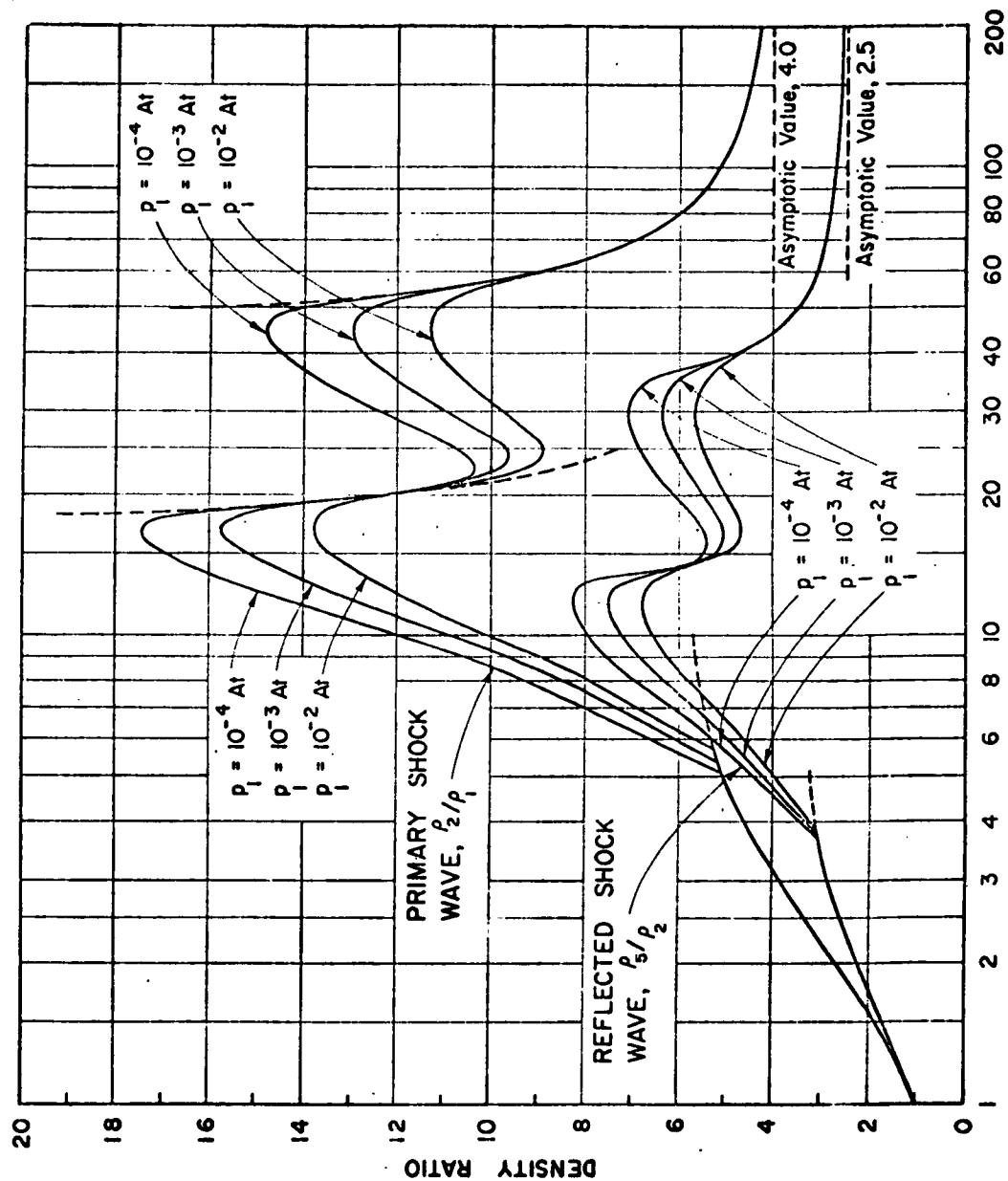


Fig. 5 Density ratios across the primary and reflected shock waves

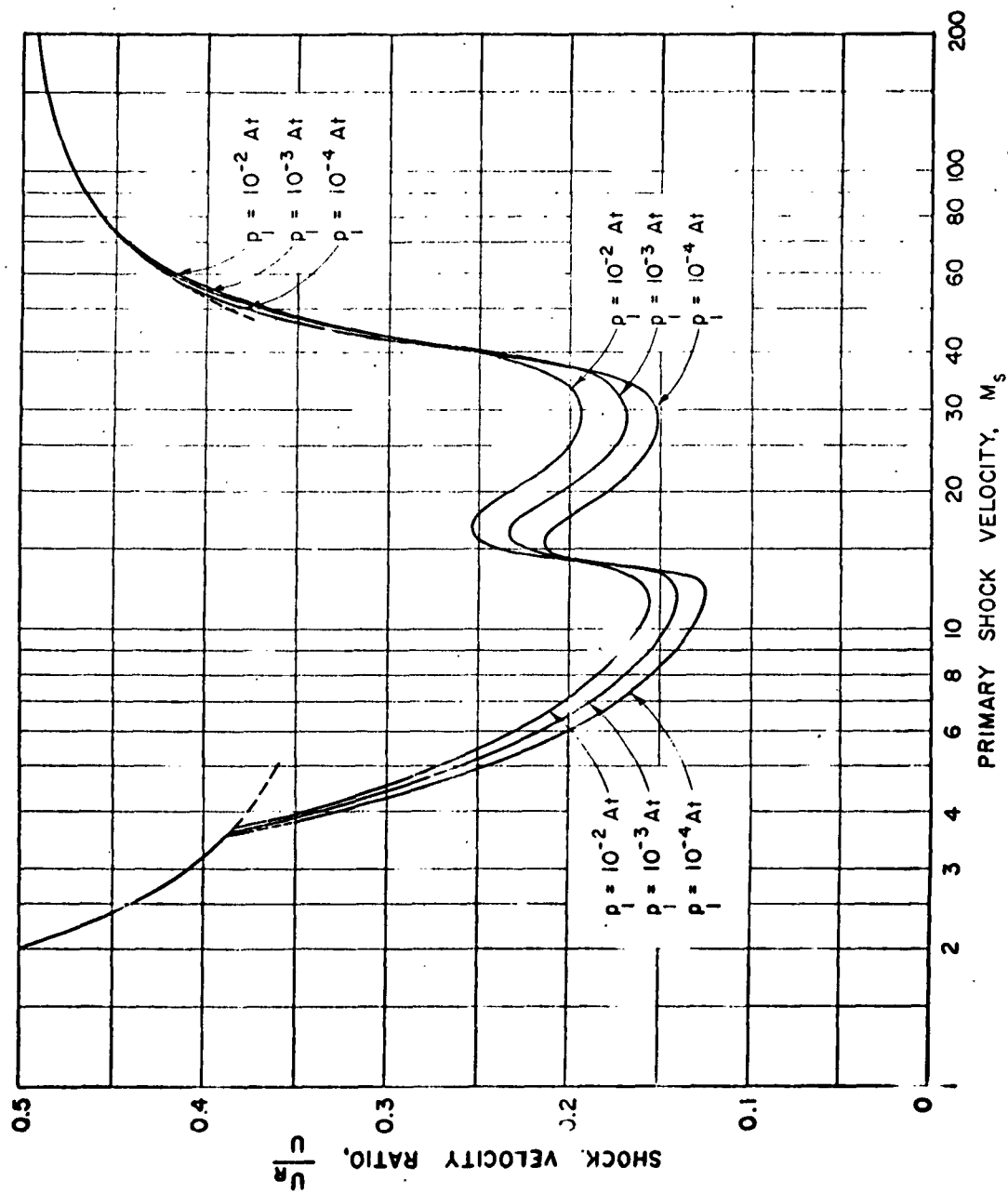


Fig. 6 Reflected shock velocity divided by the primary shock velocity

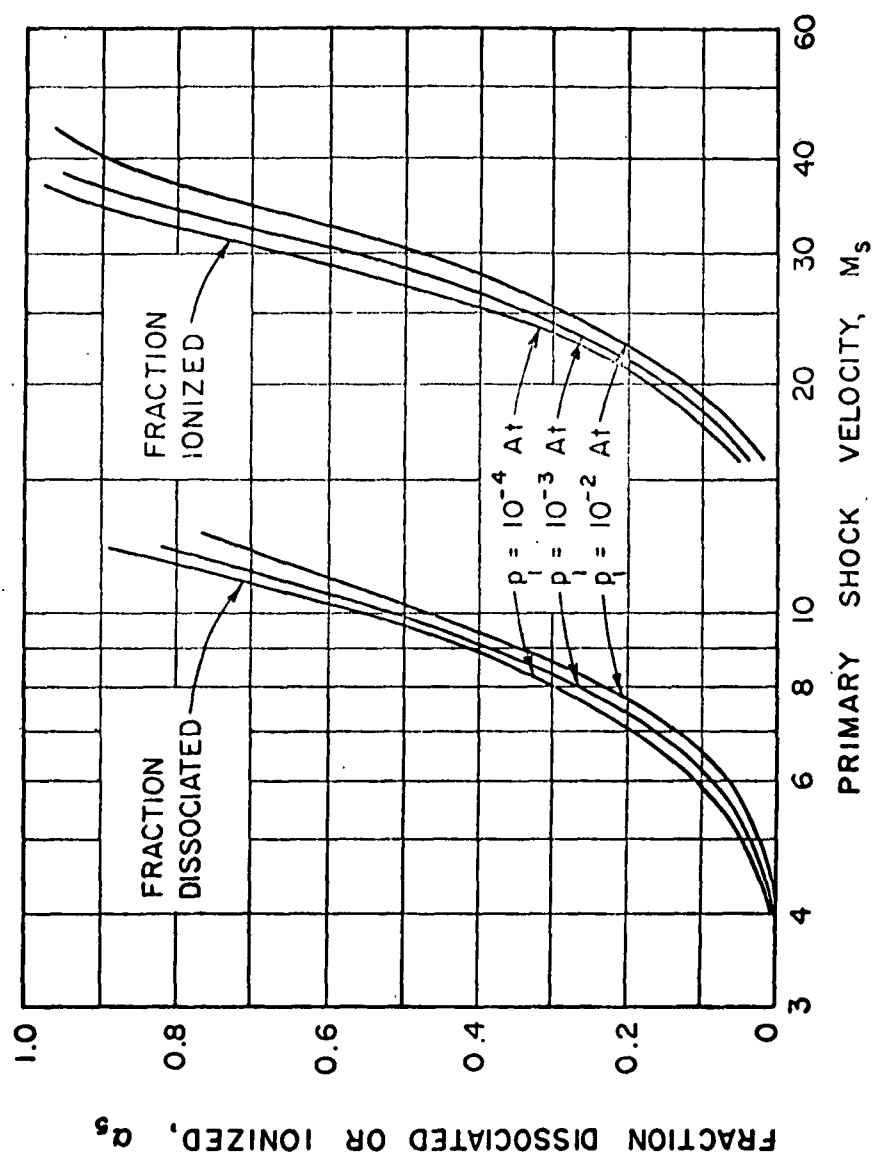


Fig. 7 Fraction ionized or dissociated behind the reflected shock wave.

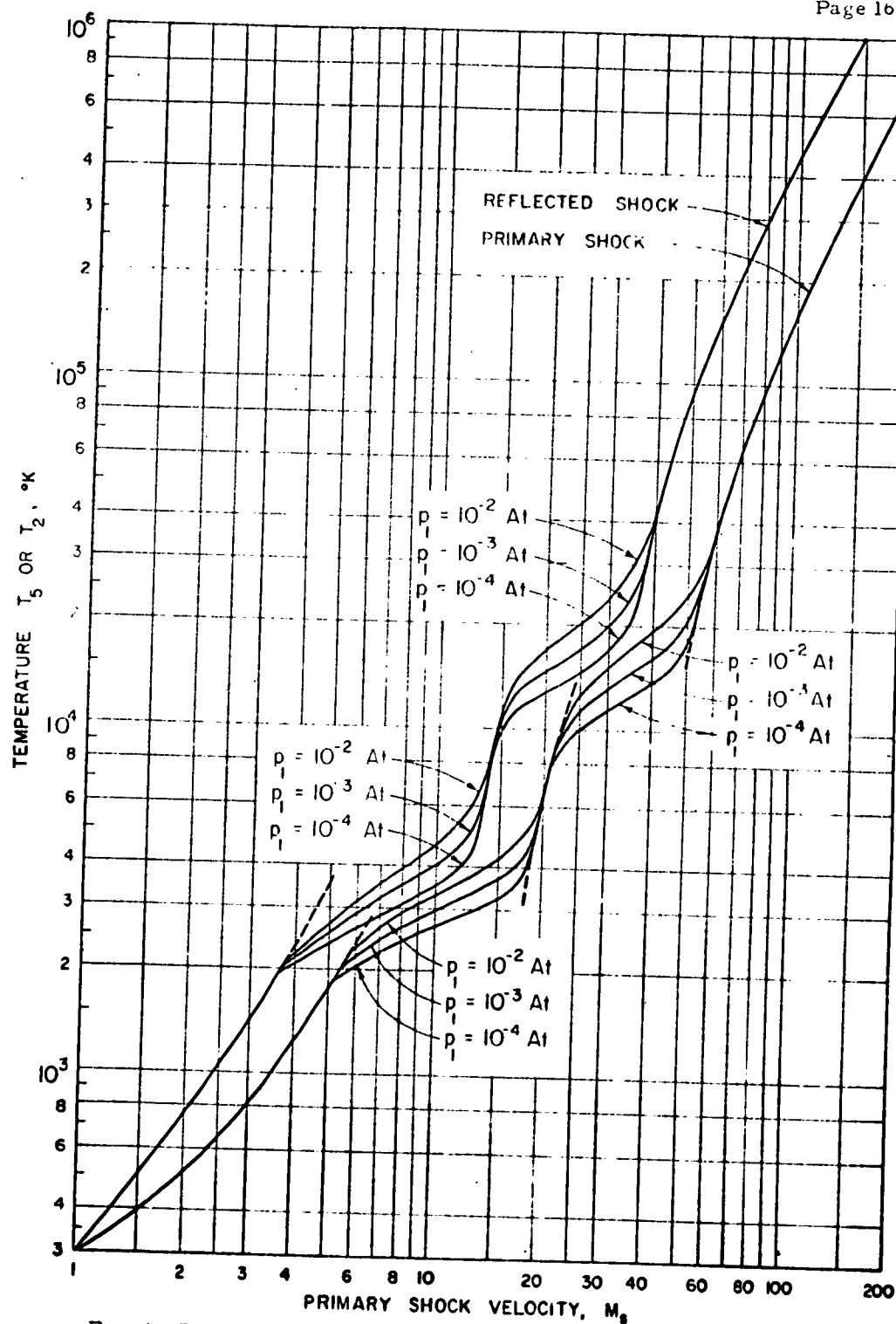


Fig. b Temperature behind primary and reflected shock waves

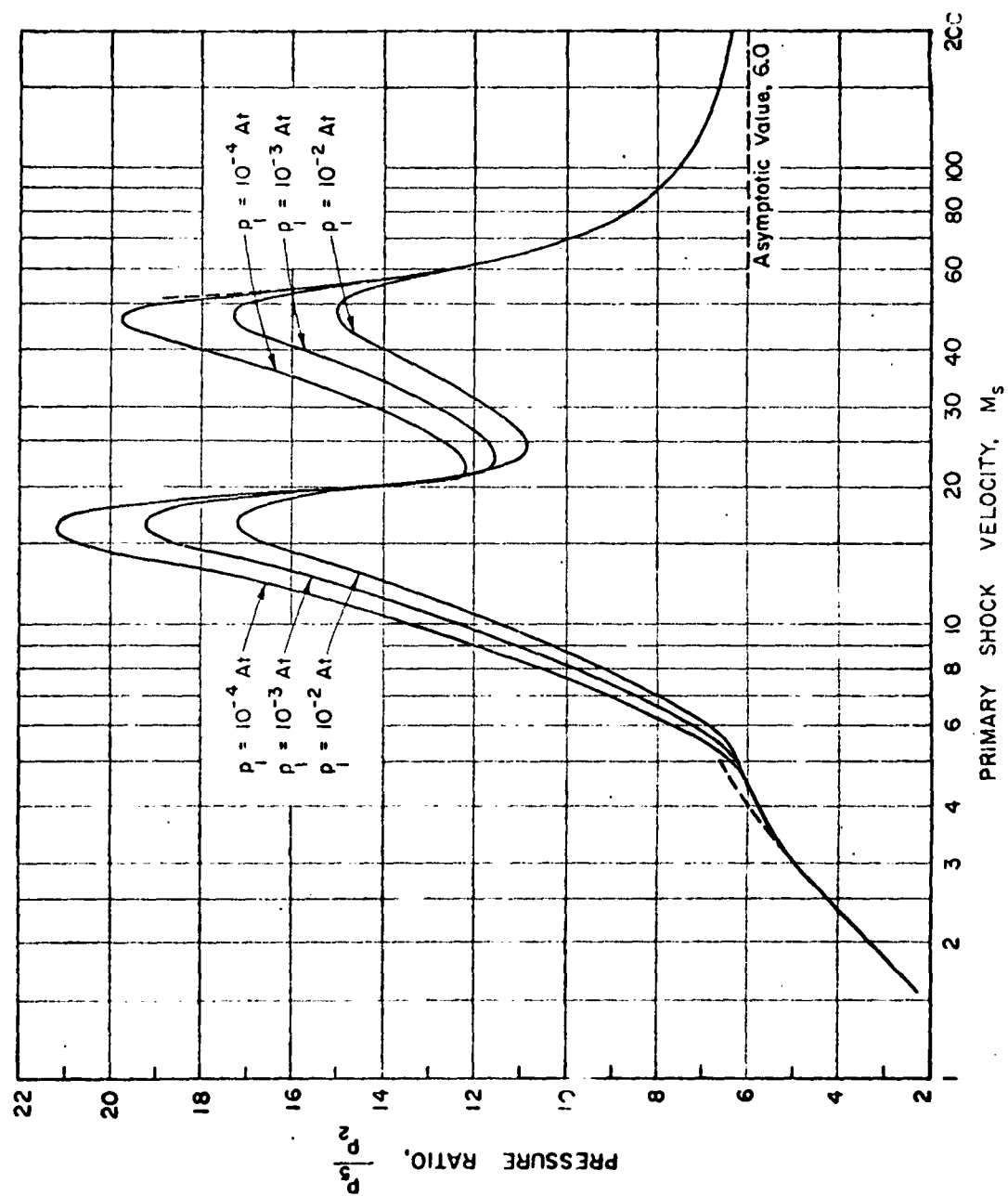


Fig. 9 Pressure ratio across the reflected shock wave